Indicators to measure multidimensional poverty

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XREAP Symposium
Universitat de Barcelona
November 25th 2016

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Income poverty

- Following Sen (1976)

1. Well-being measurement

2. Poverty threshold

3. Aggregation

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Well-being measurement

- Utility
  - Problems with adaptive preferences
- Capabilities
  - Attempt to go beyond “opulence approach”.
- Income/Consumption
  - The standard and most widely used approach in empirical research.

Poverty thresholds

- Absolute thresholds
  - Minimum / basic needs approach
- Relative thresholds
  - Reference group
- Weakly relative thresholds (Ravallion and Chen).

Aggregation

- Huge literature on poverty measures
- Foster-Greer-Thorbecke (FGT): the most popular family of indices

$$P_\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(\max\left\{0, \frac{z - x_i}{z}\right\}\right)^\alpha$$

When α=0 → Headcount ratio (H)

When α=1 → Poverty gap index

When α=2 → Inequality sensitive

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Multidimensional poverty (I)

- Individual's well-being is conceptualized taking several attributes at the same time. Grounded in Sen's Capability Approach.
- Functionings vs Capabilities

Multidimensional poverty (II)

- Lots of **additional** implementation problems
  - List of functionings to be included
  - Commensurability
  - Data availability
  - Identification of the poor
  - Aggregation
    - Combining several dimensions at the same time
    - Weights
    - Relationship between pairs of different variables

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Structure of the presentation

- Introduction ✓
- Identification
- Aggregation
- Empirical examples
- Conclusions

Identification of the poor

- Essential for the success of any poverty eradication program.
- Relatively simple in the single dimensional case (draw a poverty line…).
- **Unsatisfactorily addressed in the multidimensional (MD) case.**

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Existing approaches in the MD case

Separate distributions

↓

Indicator dashboard

Indicator dashboard

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Indicator dashboard

Ignores joint distribution, fails to identify the multiply deprived.

Existing approaches in the MD case

Separate distributions

Indicator dashboard

Joint distribution

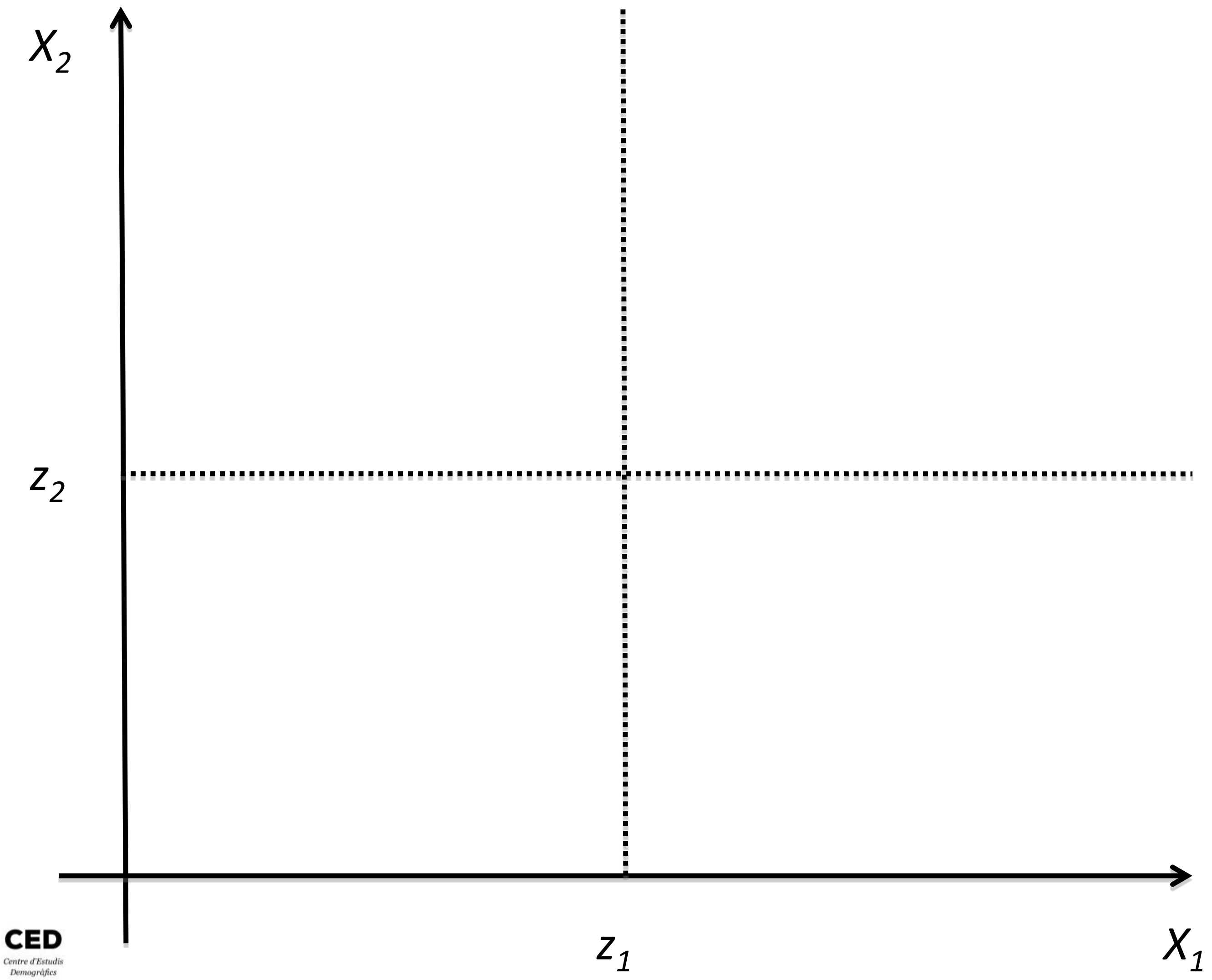
Poverty frontier (work in the achievements space)

Multiple Deprivations (work in the deprivations space)

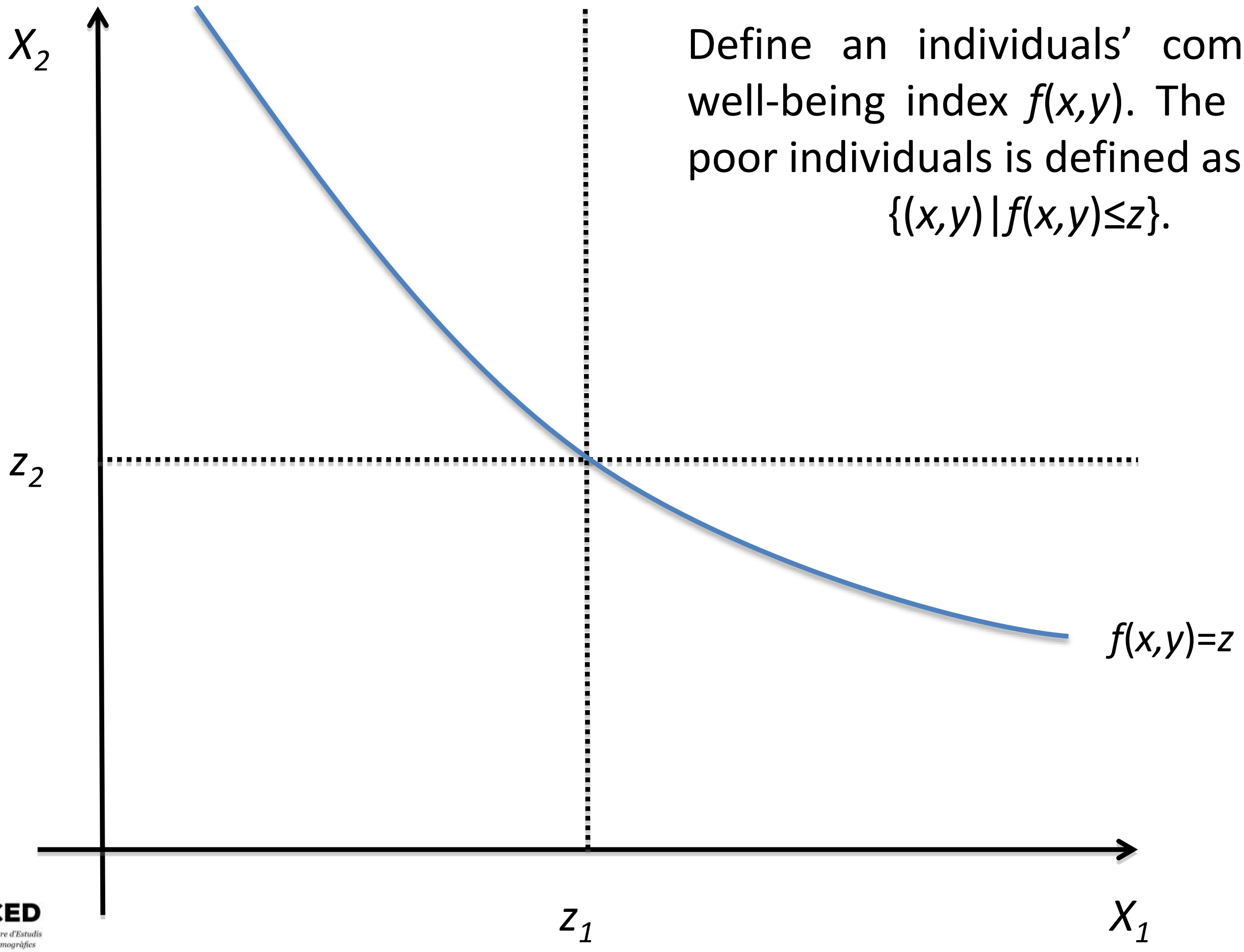
Counting approaches

- Union approach
- Intersection approach
- Intermediate approach

Joint distribution: Who is poor?



Poverty Frontier



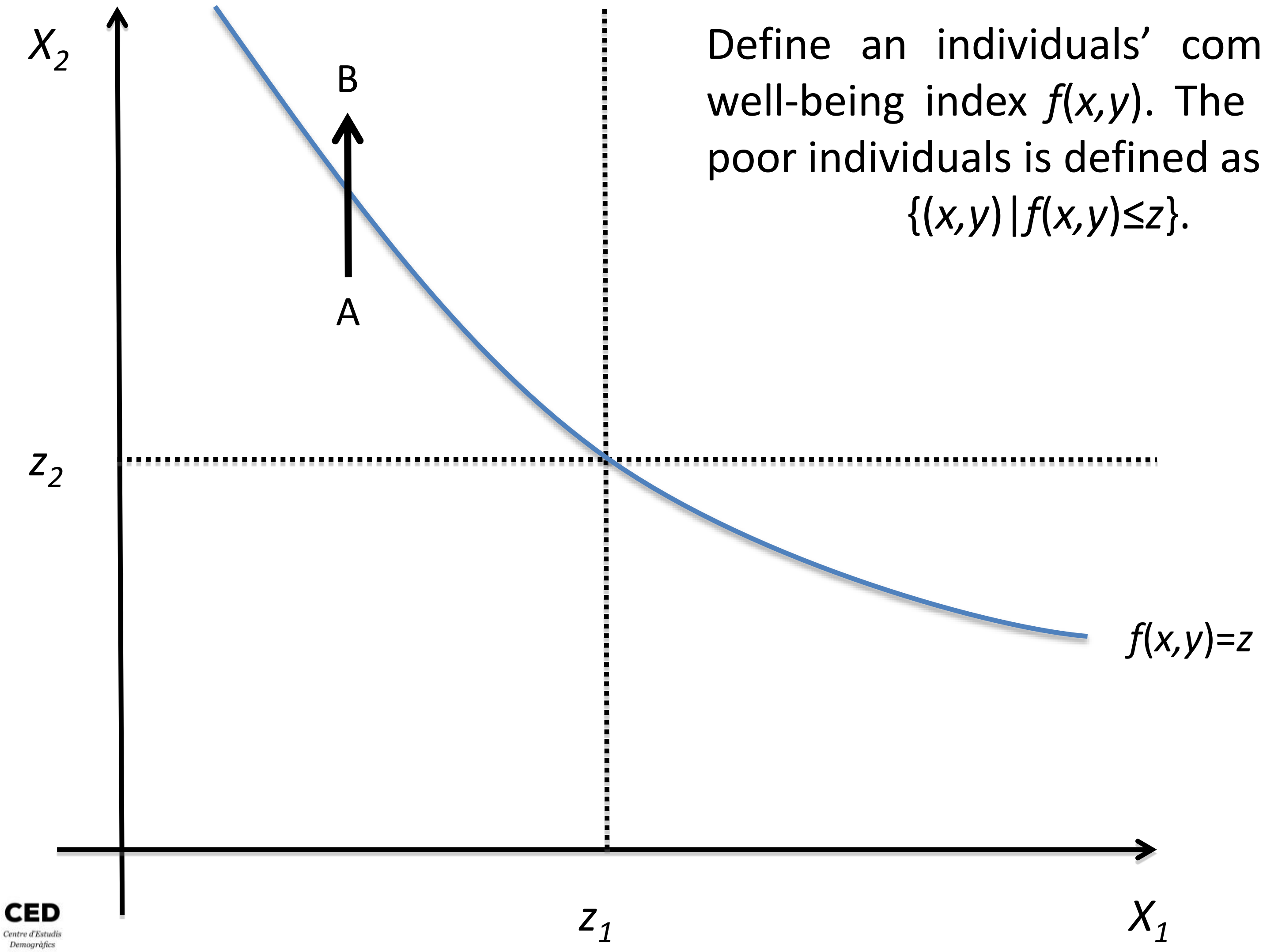
Define an individuals' composite well-being index $f(x,y)$. The set of poor individuals is defined as

$$\{(x,y)\,|\,f(x,y)\leq z\}.$$

Poverty frontier

- Reduces the multidimensional measure to a single-dimensional one.
- One can pull out of poverty individuals by increasing some non-deprived attributes, while keeping fixed the ones in which they are deprived.

Poverty Frontier



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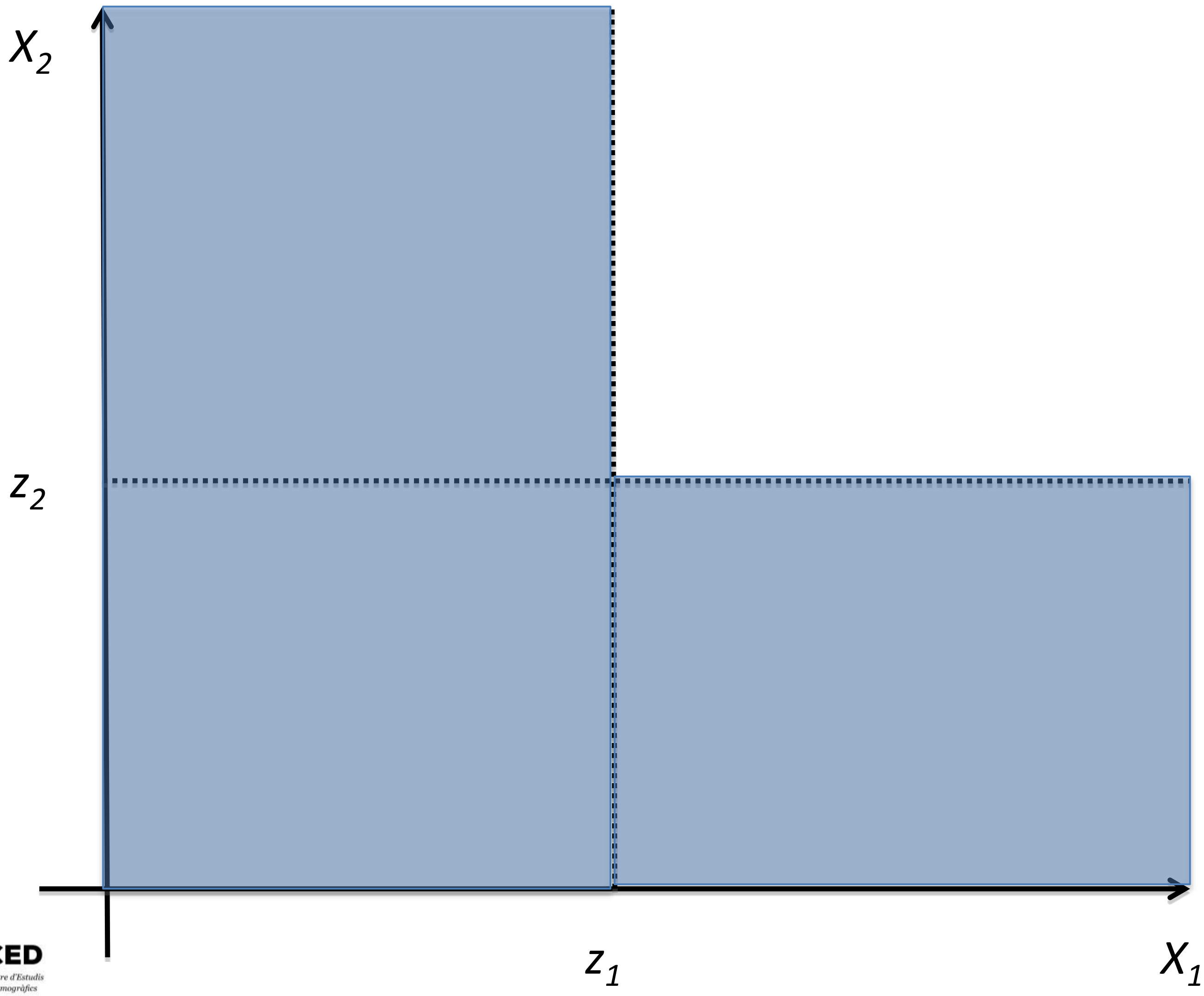
Counting approaches: Union



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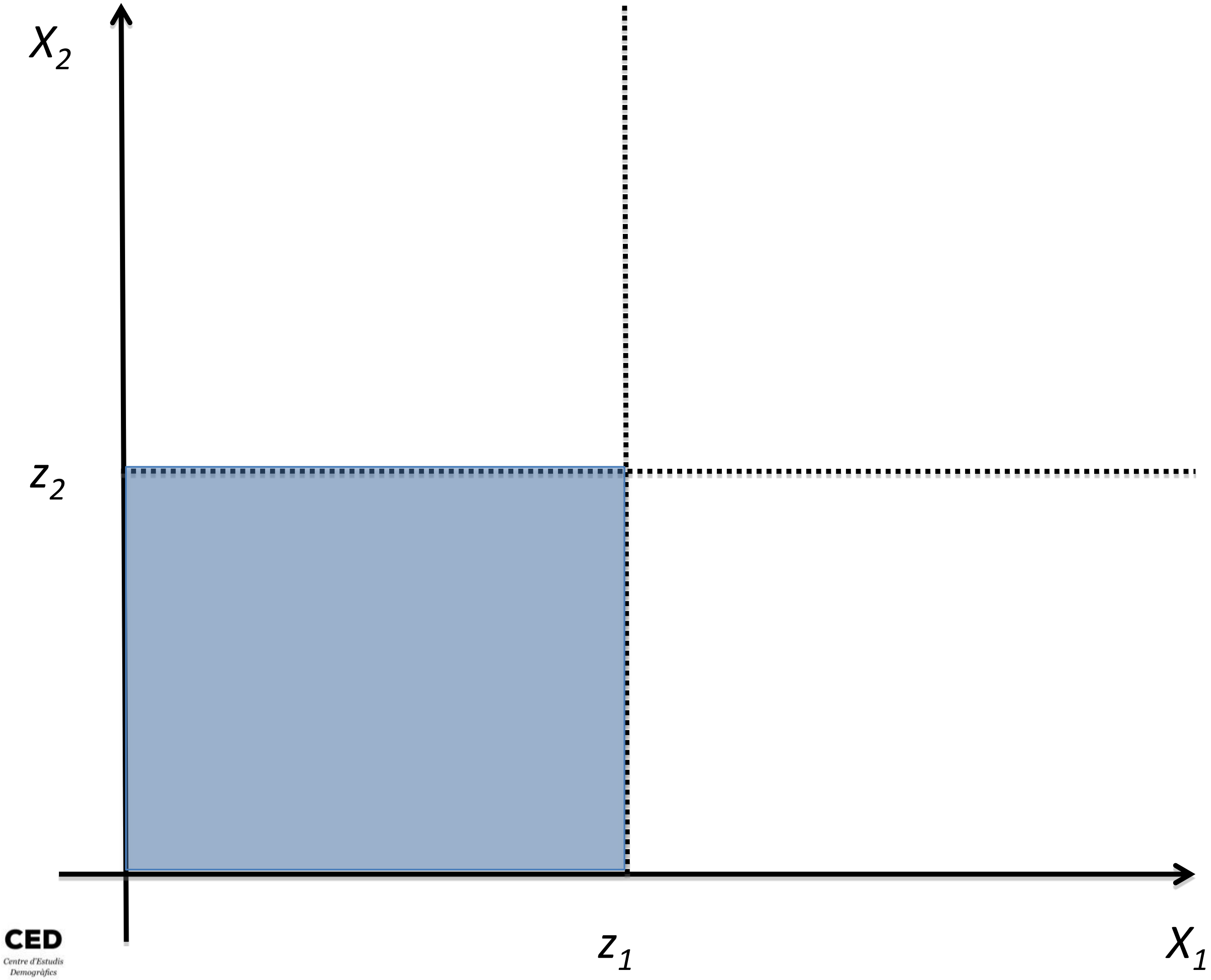
Counting approaches: Intersection



General counting approach

- Assume there are *d* dimensions, each of which with the corresponding poverty threshold z_j. We can **count** the number of dimensions in which an individual '*i*' is deprived (c_i).
- The counting approach fixes a number *k* ($1 \leq k \leq d$) and an individual '*i*' is labeled as 'poor' whenever $c_i \geq k$.
  - If k=1: Union approach
  - If k=d: Intersection approach

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Counting approach

- State-of-the-art methodology in multidimensional poverty measurement.

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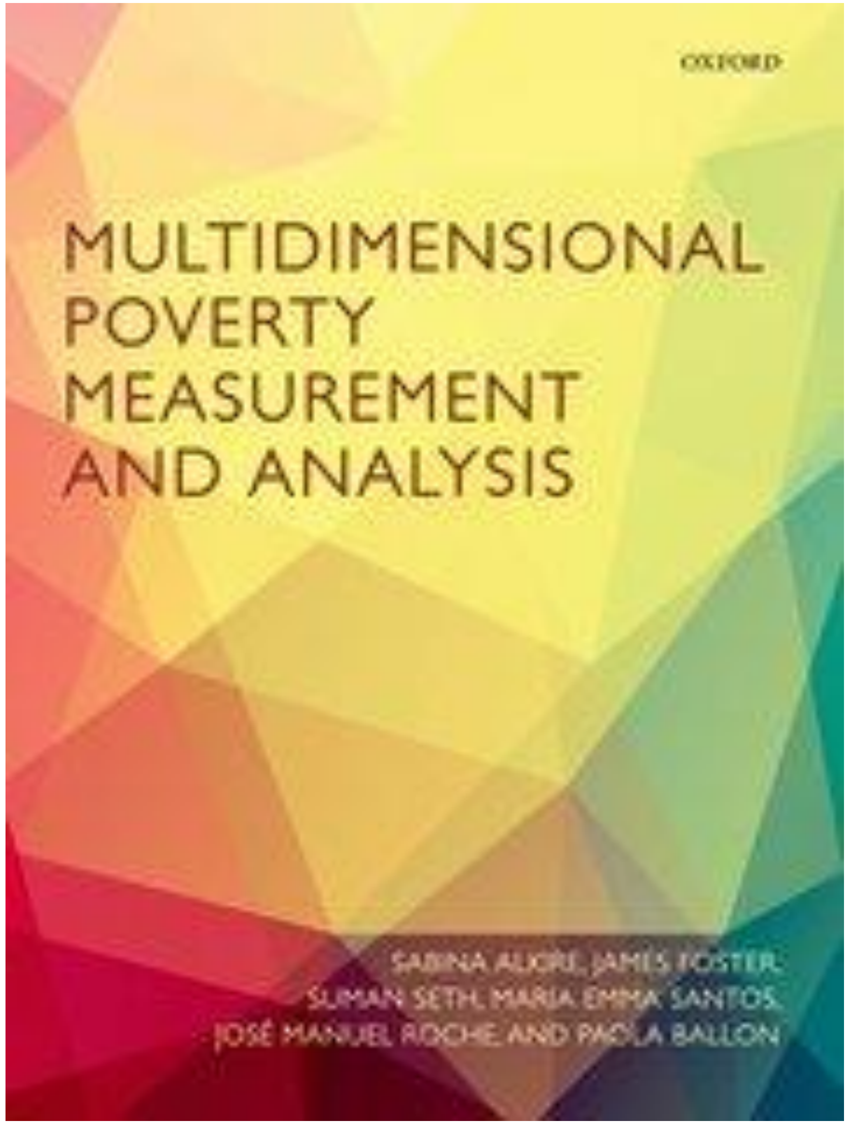

Oxford University Press 2015

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Counting approach

- State-of-the-art methodology in multidimensional poverty measurement.
- Deprivations are stacked together no matter how as long as their (weighted) sum adds up to a certain threshold (*k*).
- For instance: If *d*=4 ({A,B,C,D}),*k*=2 and equal weights apply, anyone deprived in any two dimensions is "poor":

{AB, AC, AD, BC, BD, CD}

Counting approach

- The counting approach fails to take into consideration the nature of the variables one is dealing with.
- It is related to the **Non-Preference Based** axiomatic literature on freedom (Pattanaik and Xu 1990).
- It ignores eventual relationships and interactions between different groups of variables (complementarity / substitutability issues).

Aggregation: the AF approach

- Generalization of the FGT index to the multidimensional context.
  - M_0
  - M_1
  - M_2
- Flexible identification methods.
- Can be used with ordinal data (M_0).

Aggregation

Paper	Notation	Formula	Range	MD Poverty Index
Tsui (2002)	g_{ij}^{T1}	$z_j / Min\{x_{ij}, z_j\}$	$R_{T1} = [1, +\infty)$	$\frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j=1}^{k} \left(g_{ij}^{T1}\right)^{\alpha_j} - 1 \right]$
Tsui (2002)	g_{ij}^{T2}	$\ln(z_j / Min\{x_{ij}, z_j\})$	$R_{T2} = [0, +\infty)$	$\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} \delta_j g_{ij}^{T2}$
Tsui (2002)	g_{ij}^{T3}	$z_j - Min\{x_{ij}, z_j\}$	$R_{T3} = [0, +\infty)$	$\frac{1}{n} \sum_{i=1}^{n} \left(\prod_{j=1}^{k} e^{r_j g_{ij}^{T3}} - 1 \right)$
Tsui (2002)	g_{ij}^{T4}	$z_j - Min\{x_{ij}, z_j\}$	$R_{T4} = [0, +\infty)$	$\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} c_j g_{ij}^{T4}$
B&C (2003)	g_{ij}^{BC}	$Max\left\{ \frac{z_j - x_{ij}}{z_j}, 0 \right\}$	$R_{BC} = [0, 1]$	$\frac{1}{n} \sum_{i=1}^{n} F \left(\left[\sum_{j=1}^{k} w_j \left(g_{ij}^{BC}\right)^{\theta} \right]^{1/\theta} \right)$
C&D&S (2008)	g_{ij}^{W}	$\ln(z_j / Min\{x_{ij}, z_j\})$	$R_{W} = [0, +\infty)$	$\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} g_{ij}^{W}$
A&F (2011)	g_{ij}^{AF}	$Max\left\{ \frac{z_j - x_{ij}}{z_j}, 0 \right\}$	$R_{AF} = [0, 1]$	$\frac{1}{nk} \sum_{i \in P} \sum_{j=1}^{k} \left(g_{ij}^{AF}\right)^{\alpha}$

Aggregation

Paper	Notation	Formula	Range	MD Poverty Index
Tsui (2002)	g_{ij}^{T1}	$z_j/Min\{x_{ij}, z_j\}$	$R_{T1} = [1, +\infty)$	$\frac{1}{n}\sum_{i=1}^{n}\left[\prod_{j=1}^{k}(g_{ij}^{T1})^{\alpha_j} - 1\right]$
Tsui (2002)	[illegible]	[illegible]	[illegible]	[illegible]
Tsu[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Tsu[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
B&C [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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All pairs of attributes are either complements or substitutes

Decomposability

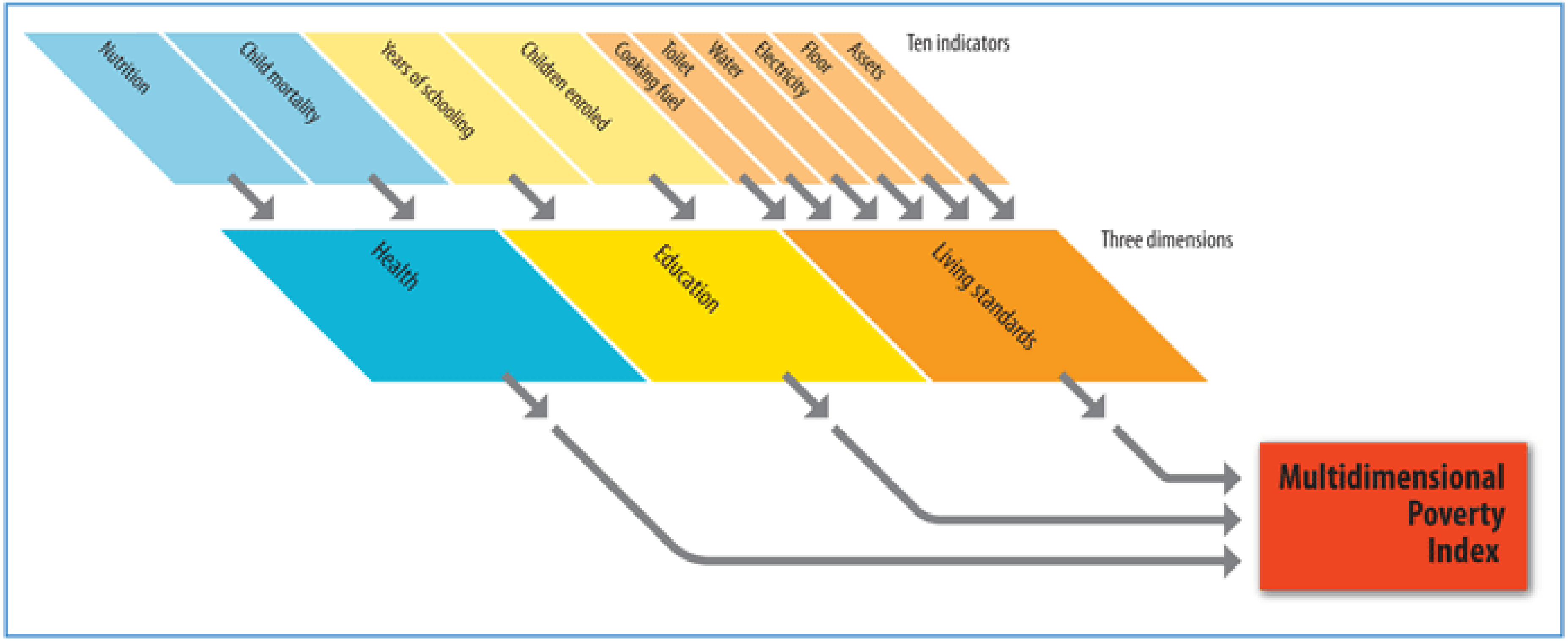
- Useful to know the contribution of each dimension to overall poverty.
- Limits the criticism against composite index approaches.
- Decomposability is at odds with non-trivial dependency structures.

Empirical examples

Human Development Report 2010

FIGURE 5.7 **Components of the Multidimensional Poverty Index**

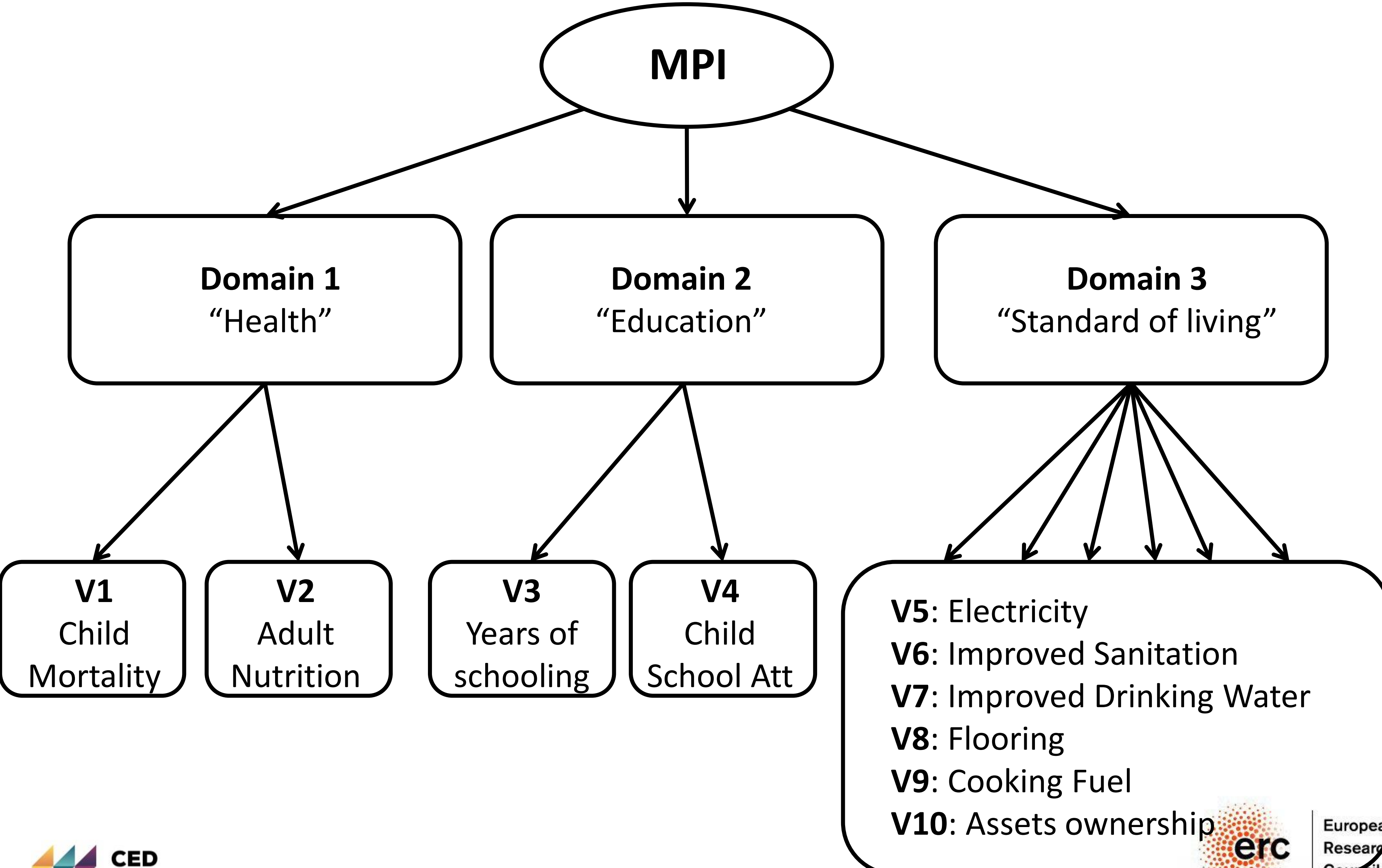
MPI—three dimensions and 10 indicators



Note: The size of the boxes reflects the relative weights of the indicators.

Source: Alkire and Santos 2010.

Empirical Example: UNDP's MPI



Dimensions and deprivations

Dimensions of poverty	Indicator	Deprived if…	Weight
Education	Years of Schooling	No household member has completed five years of schooling.	1/6
	Child School Attendance	Any school aged child is not attending school up to class 8.	1/6
Health	Child Mortality	Any child has died in the family.	1/6
	Nutrition	Any adult for whom there is nutritional information is malnourished.	1/6
Living Standard	Electricity	The household has no electricity.	1/18
	Improved Sanitation	The household's sanitation facility is not improved (according to MDG guidelines), or it is improved but shared with other households.	1/18
	Improved Drinking Water	The household does not have access to improved drinking water (according to MDG guidelines) or safe drinking water is more than a 30-minute walk from home, roundtrip.	1/18
	Flooring	The household has a dirt, sand or dung floor.	1/18
	Cooking Fuel	The household cooks with dung, wood or charcoal.	1/18
	Assets ownership	The household does not own more than one radio, TV, telephone, bike, motorbike or refrigerator and does not own a car or truck.	1/18

Results (I)

Table 3: Summary MPI and income poverty estimates by UN regions

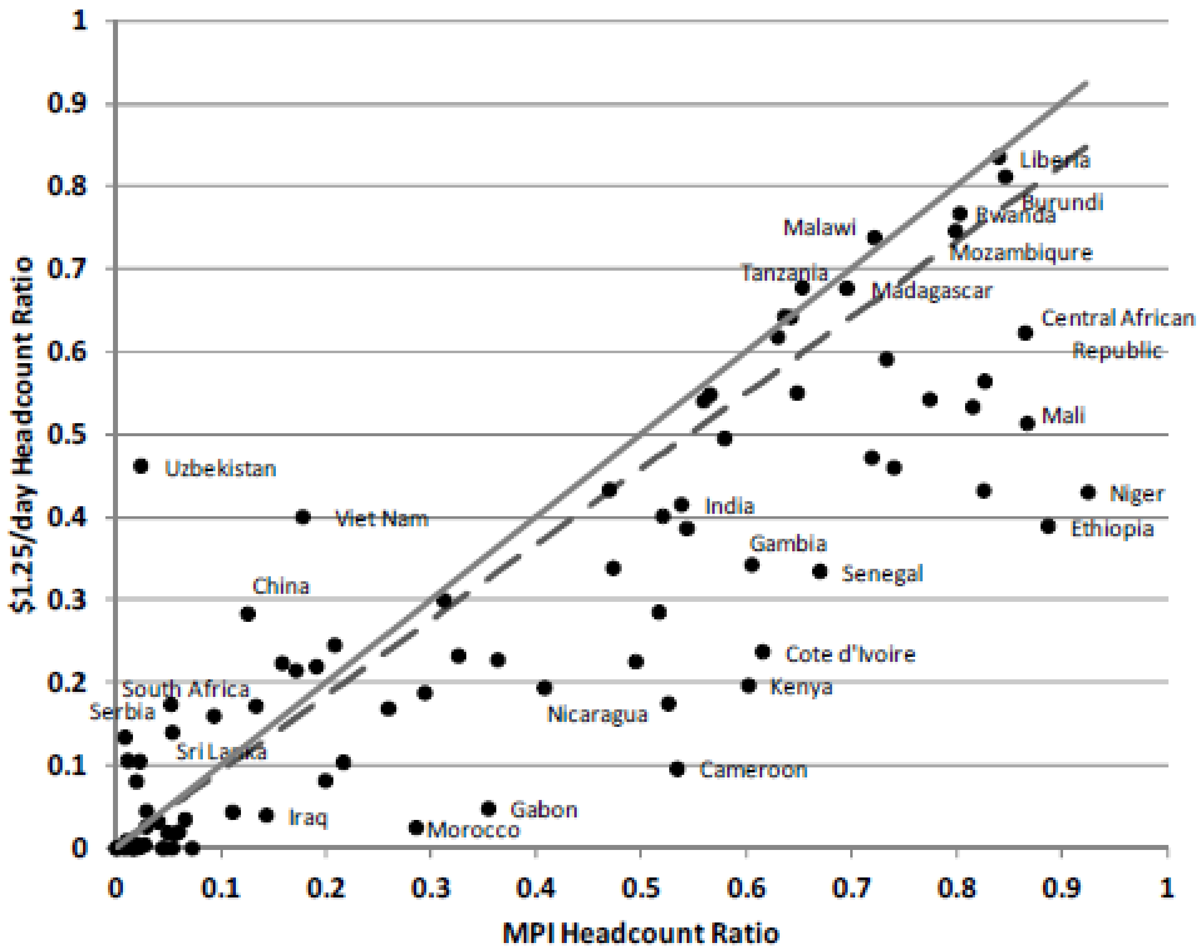
Region of the World	Total Pop. (millions)	H	A	MPI	MPI poor pop. (millions)	$1.25/day poor	$1.25/day poor pop. (millions)	$2/day poor	$2/day poor pop. (millions)
CEE and CIS	398.3	0.029	0.394	0.011	11.4	0.045	18.0	0.110	43.8
LAC	491.8	0.154	0.419	0.064	75.6	0.101	49.8	0.200	98.2
EAP	1864.5	0.146	0.457	0.066	271.4	0.265	494.4	0.498	927.7
AS	212.7	0.179	0.508	0.091	38.0	0.038	8.1	0.194	41.2
SA	1531.0	0.532	0.526	0.280	814.9	0.402	615.4	0.741	1133.8
SSA	703.7	0.647	0.577	0.374	455.5	0.486	342.3	0.705	496.2
Total 104 countries	**5202.1**	**0.320**	**0.522**	**0.167**	**1666.8**	**0.294**	**1528.0**	**0.527**	**2741.0**

Note: Pop. is Population, expressed in millions. H, A, MPI, $1.25/day poor and $2/day poor are all proportions.

CEE and CIS: Central and Eastern Europe and the Commonwealth of Independent States. LAC: Latin America and the Caribbean. EAP: East Asia and the Pacific. AS: Arab States. SA: South Asia. SSA: Sub-Saharan Africa.

Results (II)

Figure 3: MPI poor headcount ratio vs. $1.25/day poor headcount ratio



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AROPE

- Composite index of ‘risk-of-poverty-and-social-exclusion’ in European countries.
- Three components
  - Income poverty (below 60% Median)
  - Low work intensity (work less than 20% of total potential)
  - Material deprivation (not able to afford 4 out of 9 basic items).
- Union approach

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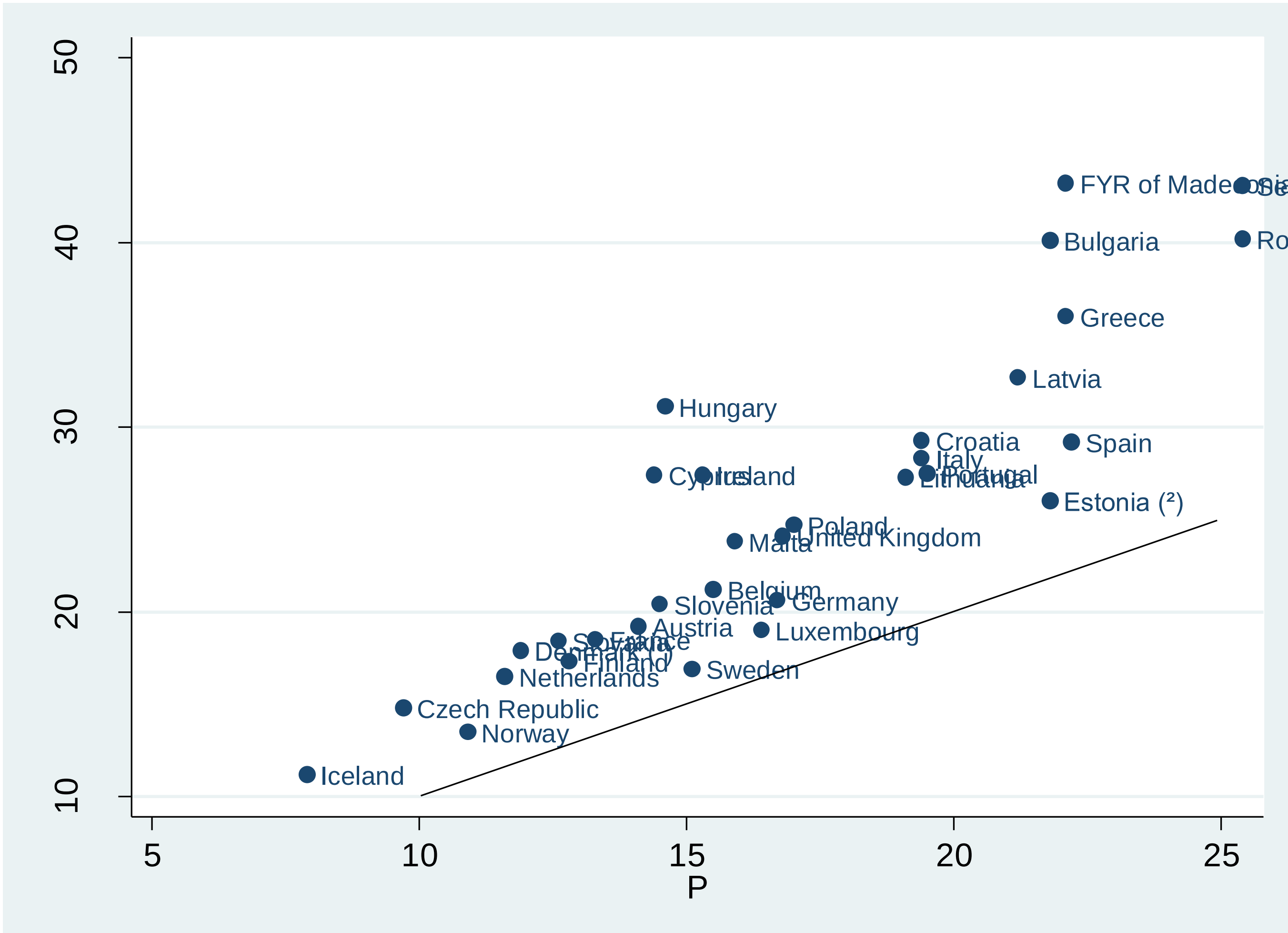
AROPE across European countries (Year 2014)



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Summary and conclusions (1)

- MDP measures offer a more complete / comprehensive perspective of well-being deprivation.
- Yet, haunted by many technical problems
  - Choice of relevant dimensions?
  - Data availability
  - Identification method?
  - Aggregation method?

Summary and conclusions (2)

- Trade-offs variability across dimension pairs.
- Current methods assume constant elasticity of substitution among **all dimension pairs**.
- **Crucial implications** for poverty eradication programs.

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Notation and definitions

- N: Set of individuals $|N|=n$.
- D: Set of dimensions $|D|=d$.
- For each individual *i* we consider her *achievement vector*

$$\boldsymbol{y}_i=(y_{i1},\ldots,y_{id})$$

(where $y_{ij}\in I_j$) and a vector of poverty thresholds $\boldsymbol{z}=(z_1,\ldots,z_d)$.

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Identification functions

$$\zeta : (I_1 \times \ldots \times I_d) \times (I_1 \times \ldots \times I_d) \rightarrow \{0,1\}$$

$\zeta(\boldsymbol{y}_i, \boldsymbol{z})=1$ if person *i* is poor and 0 otherwise.

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$\zeta(\mathbf{y}_i, \mathbf{z}) = 1$ if person *i* is poor and 0 otherwise.

Let $X^d := \{0,1\}^d$. We decompose ζ as $\zeta = \rho \circ \omega$

$$\omega : (I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \rightarrow X^d$$

(within dimensions identification function)

$$\rho : X^d \rightarrow \{0, 1\}$$

(between dimensions identification function)

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Notation and definitions

Set of **deprivation profiles:** $X^d = \{0,1\}^d$

Set of **identification functions**

$$\Omega_d := \{\rho | \rho : X^d \rightarrow \{0,1\}\}$$

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Set of **identification functions**

$$\Omega_d := \{\rho | \rho : X^d \to \{0,1\}\}$$

Set of **poor profiles**

$$P_\rho := \{\mathbf{x} \in X^d | \rho(\mathbf{x}) = 1\} = \rho^{-1}(1)$$

Set of **non-poor profiles**

$$R_\rho := \{\mathbf{x} \in X^d | \rho(\mathbf{x}) = 0\} = \rho^{-1}(0) = X^d \setminus P_\rho$$

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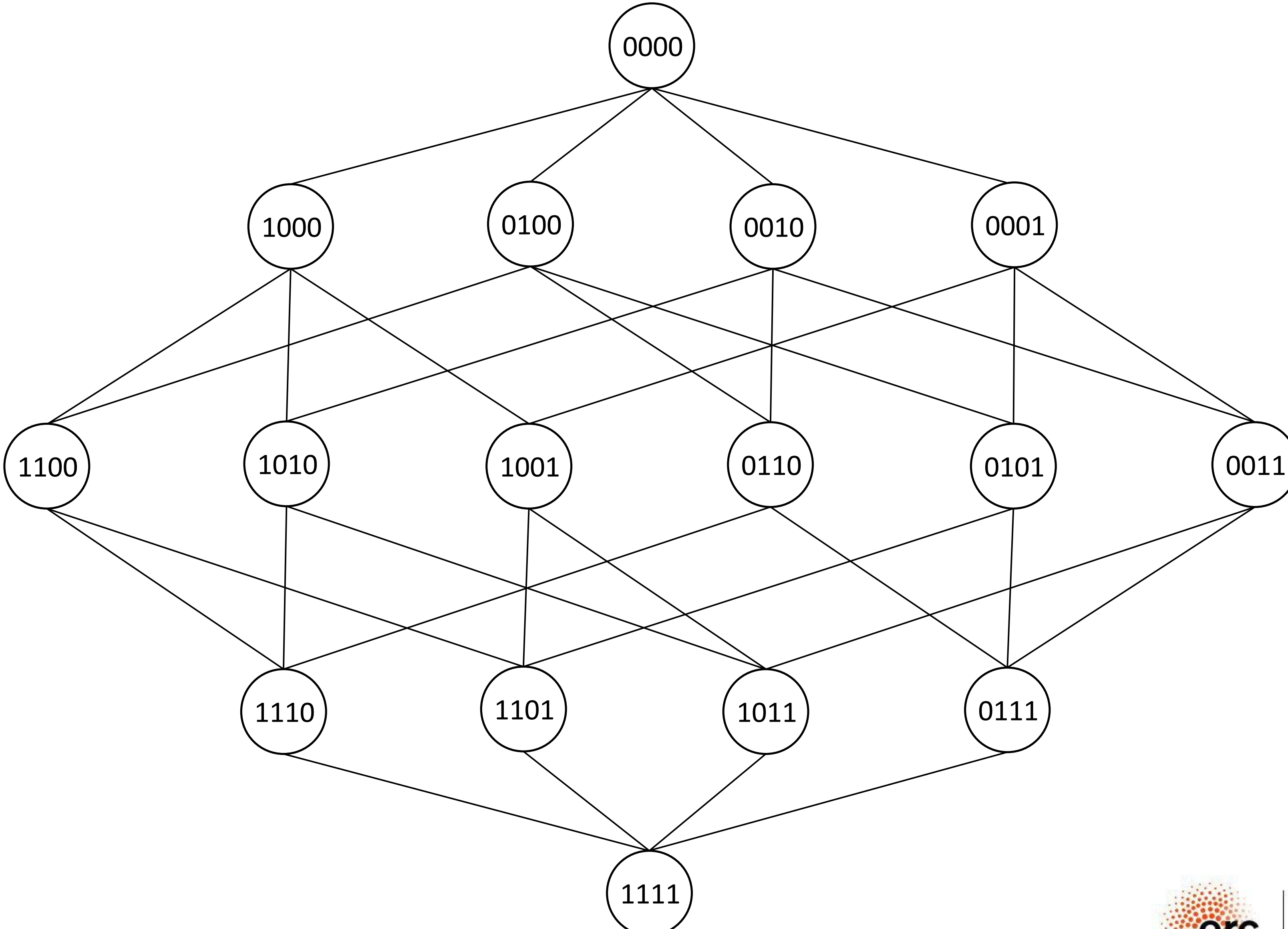
Hasse diagrams



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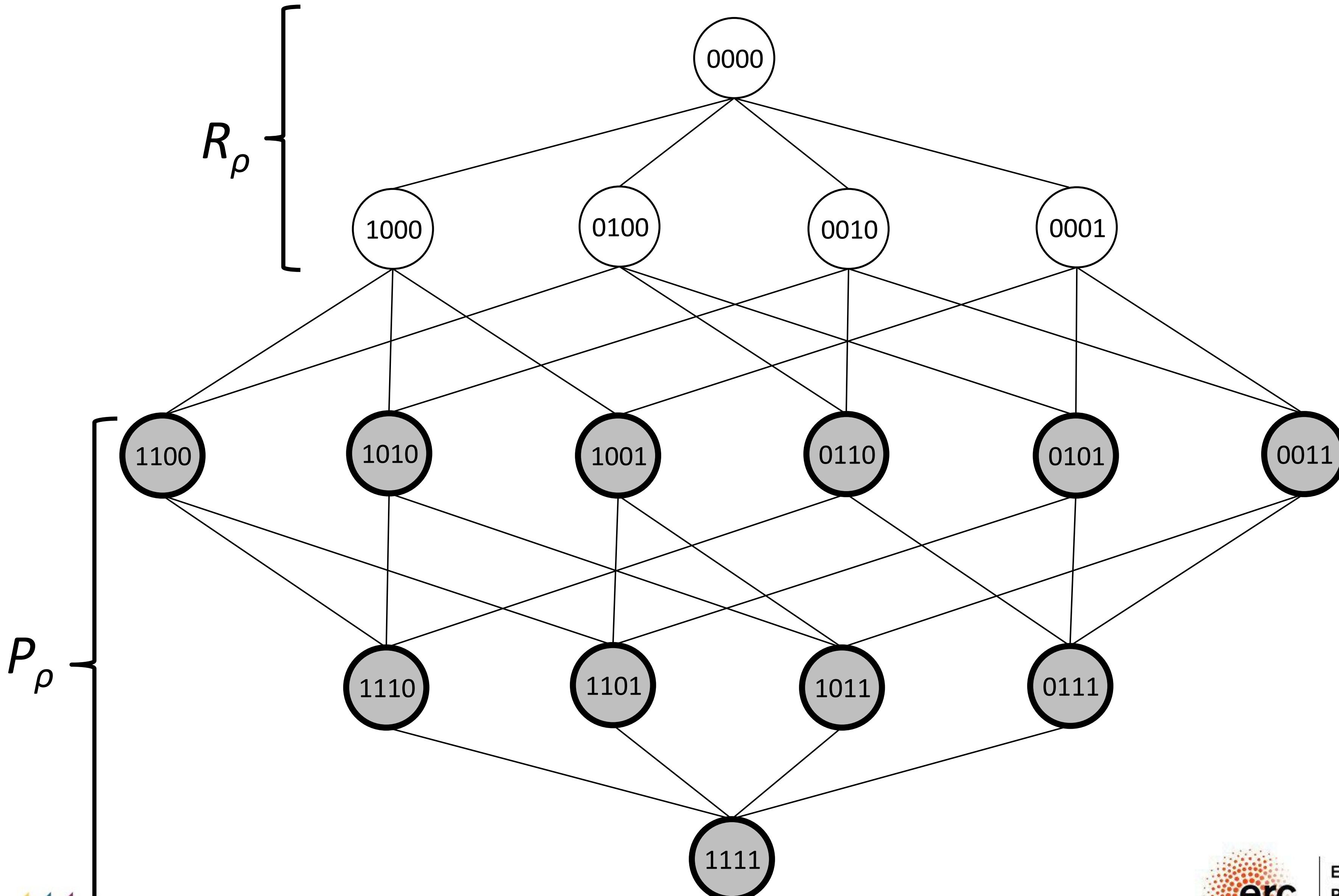

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The (weighted) counting approach

- For any $\boldsymbol{a}=(a_1,\ldots,a_d)\in\Delta_d$, $\boldsymbol{x}\in X^d$, let

$$c_{\mathbf{a}}(\mathbf{x}) = \sum_{j=1}^{j=d} a_j x_j \qquad \iota_k(s) = \left\{ \begin{array}{c} 1 \text{ if } s \geq k \\ 0 \text{ if } s < k \end{array} \right\}$$

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$$\mathcal{C}_d^{\mathbf{a}} := \{\rho \in \Omega_d | \rho(\mathbf{x}) = \iota_k(c_{\mathbf{a}}(\mathbf{x})) \text{ for some } k \in (0,1]\}$$

$$\mathcal{W}_d := \bigcup_{\mathbf{a}\in\Delta_d} \mathcal{C}_d^{\mathbf{a}}$$

Sets of identification functions belonging to the **weighted counting approach** (Alkire and Foster).

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Sets of identification functions belonging to the **weighted counting approach** (Alkire and Foster).

RQ: Does this exhaust all “reasonable” identification functions one could think of?

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Axiomatic characterization of ρ

Monotonicity (MON): Let $\mathbf{x}, \mathbf{y} \in X^d$. If one has that $\mathbf{x} \leq \mathbf{y}$, then $\rho(\mathbf{x}) \leq \rho(\mathbf{y})$ for all $\rho \in S$.

Independence (IND): Let $\mathbf{x}, \mathbf{y} \in X^d$ be two deprivation profiles such that for some dimension $i \in \{1, \ldots, d\}, x_i = y_i$. Let $\mathbf{x}', \mathbf{y}' \in X^d$ be two other deprivation profiles such that $x_j = x'_j$ and $y_j = y'_j$ for $j \neq i$ and $x'_i = y'_i$. Then $\rho(\mathbf{x}) \leq \rho(\mathbf{y})$ implies $\rho(\mathbf{x}') \leq \rho(\mathbf{y}')$ for all $\rho \in S$.

Anonymity (ANO): For any $i, j \in \{1, \ldots, d\}$ one has that $\rho(\mathbf{e}_i) = \rho(\mathbf{e}_j)$ for all $\rho \in S$.

Non-triviality (NTR): ρ is a non-constant function for all $\rho \in S$.

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Non-triviality (NTR): ρ is a non-constant function for all $\rho \in S$.

Theorem 1: Let $S \subseteq \Omega_d$. One has that the different $\rho \in S$ satisfy MON, IND, ANO and NTR if and only if $S = C_d^{\mathbf{1}/d}$.

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A definition and an axiom

Definition 1: Consider two hypothetical societies, each with $m > 1$ individuals, with deprivation profiles $(\mathbf{x}_1, \ldots, \mathbf{x}_m)$, $(\mathbf{y}_1, \ldots, \mathbf{y}_m)$. We say that these two societies are *equivalent* if for each dimension $j \in \{1, \ldots, d\}$ the number of individuals that are deprived in that dimension is the same in both societies, that is: $\sum_{i=1}^{i=m} x_{ij} = \sum_{i=1}^{i=m} y_{ij}\ \forall j \in \{1, \ldots, d\}$.

Compensation (COM): Consider two equivalent societies with deprivation profiles $(\mathbf{x}_1, \ldots, \mathbf{x}_m)$ and $(\mathbf{y}_1, \ldots, \mathbf{y}_m)$. Assume that $\rho(\mathbf{x}_1) \geq \rho(\mathbf{y}_1), \ldots, \rho(\mathbf{x}_{m-1}) \geq \rho(\mathbf{y}_{m-1})$ for all $\rho \in S$. Then, one must have that $\rho(\mathbf{x}_m) \leq \rho(\mathbf{y}_m)$ for all $\rho \in S$.

Characterization of the weighted case

Theorem 2: Let $S \subseteq \Omega_d$. One has that the different $\rho \in S$ satisfy MON, COM and NTR if and only if $S = C_d^{\mathbf{a}}$ for some $\mathbf{a} \in \Delta_d$.

- Monotonicity and Non-triviality seem indisputable.
- Compensation imposes separability across dimensions

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Characterization of the weighted case

Theorem 2: Let $S \subseteq \Omega_d$. One has that the different $\rho \in S$ satisfy MON, COM and NTR if and only if $S = C_d^{\mathbf{a}}$ for some $\mathbf{a} \in \Delta_d$.

- Monotonicity and Non-triviality seem indisputable.
- Compensation imposes separability across dimensions → Get rid of it.

The consistency condition

Definition 2: We say that a set of identification functions $S \subseteq \Omega_d$ satisfies the *Consistency Condition* (CC) whenever MON and NTR are satisfied. The set of all such identification functions will be denoted as $\mathcal{P}_d$.

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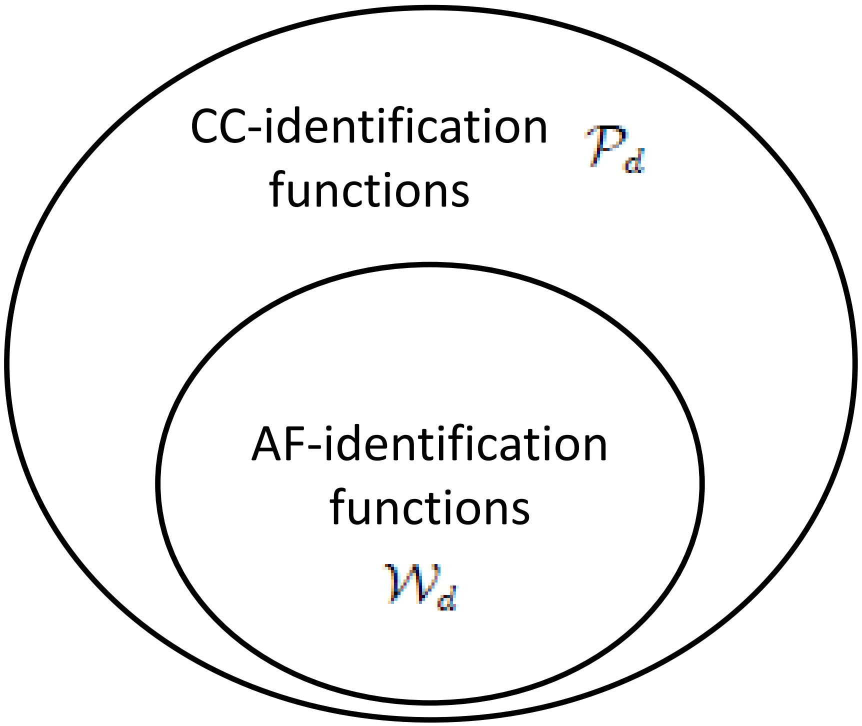
The consistency condition

Definition 2: We say that a set of identification functions $S \subseteq \Omega_d$ satisfies the *Consistency Condition* (CC) whenever MON and NTR are satisfied. The set of all such identification functions will be denoted as $\mathcal{P}_d$.

Proposition 3: If $d \in \{2, 3\}$, $\mathcal{W}_d = \mathcal{P}_d$. For any $d \geq 4$, $\mathcal{W}_d \subsetneq \mathcal{P}_d$.



Journal of Public Economics 95 (2011) 476–487

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

Counting and multidimensional poverty measurement

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ARTICLE INFO

Article history:
Received 21 October 2009
Received in revised form 18 September 2010
Accepted 4 November 2010
Available online 2 December 2010

JEL classification:
I3
I32
D63
O1
H1

Keywords:
Multidimensional poverty measurement
Capability approach
Identification
FGT measures
Decomposability
Ordinal variables

ABSTRACT

This paper proposes a new methodology for multidimensional poverty measurement consisting of an identification method ρ_k that extends the traditional intersection and union approaches, and a class of poverty measures M_α. Our identification step employs two forms of cutoff: one within each dimension to determine whether a person is deprived in that dimension, and a second across dimensions that identifies the poor by 'counting' the dimensions in which a person is deprived. The aggregation step employs the FGT measures, appropriately adjusted to account for multidimensionality. The axioms are presented as joint restrictions on identification and the measures, and the methodology satisfies a range of desirable properties including decomposability. The identification method is particularly well suited for use with ordinal data, as is the first of our measures, the adjusted headcount ratio M_0. We present some dominance results and an interpretation of the adjusted headcount ratio as a measure of unfreedom. Examples from the US and Indonesia illustrate our methodology.

Journal of Public Economics 95 (2011) 476–487

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

9. Illustrative examples

We now illustrate the measurement methodology and its variations using data from the United States and Indonesia.

9.1. United States

To estimate multidimensional poverty in the US we use data from the 2004 National Health Interview Survey[35] on adults aged 19 and above ($n = 45{,}884$). We draw on four variables: (1) income measured in poverty line increments and grouped into 15 categories, (2) self-reported health, (3) health insurance, and (4) years of schooling. For this

Counting a

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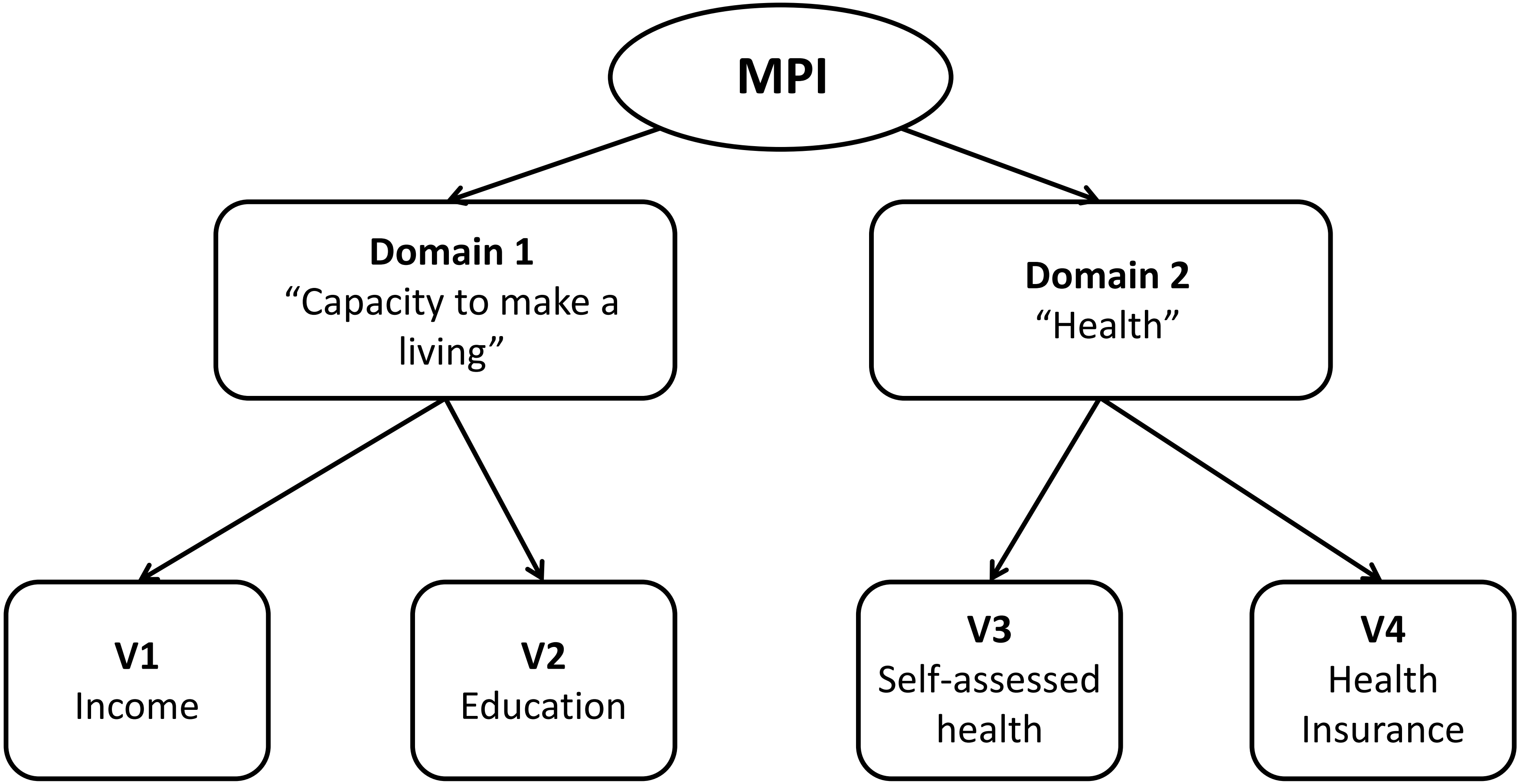
An illustrative example



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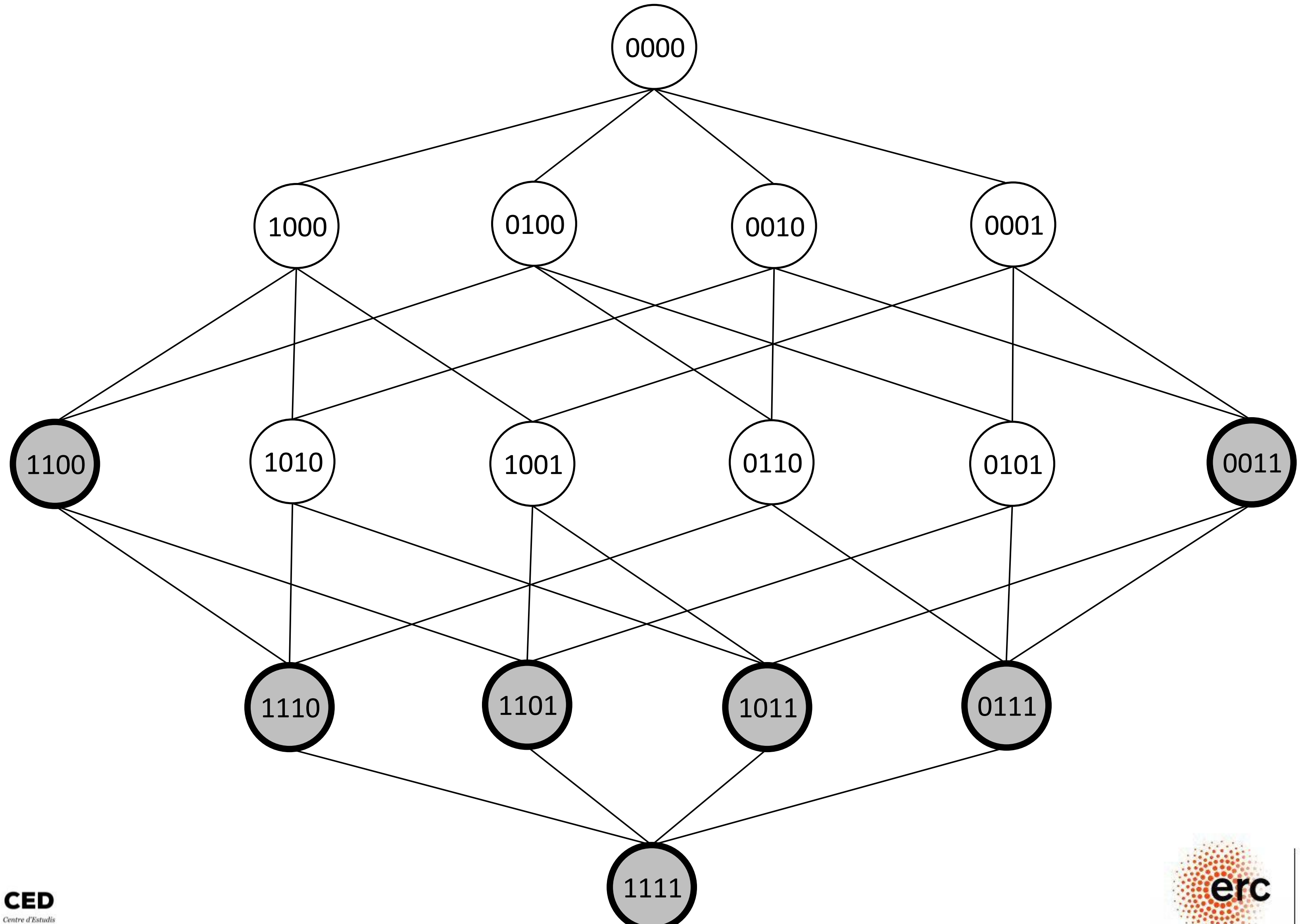
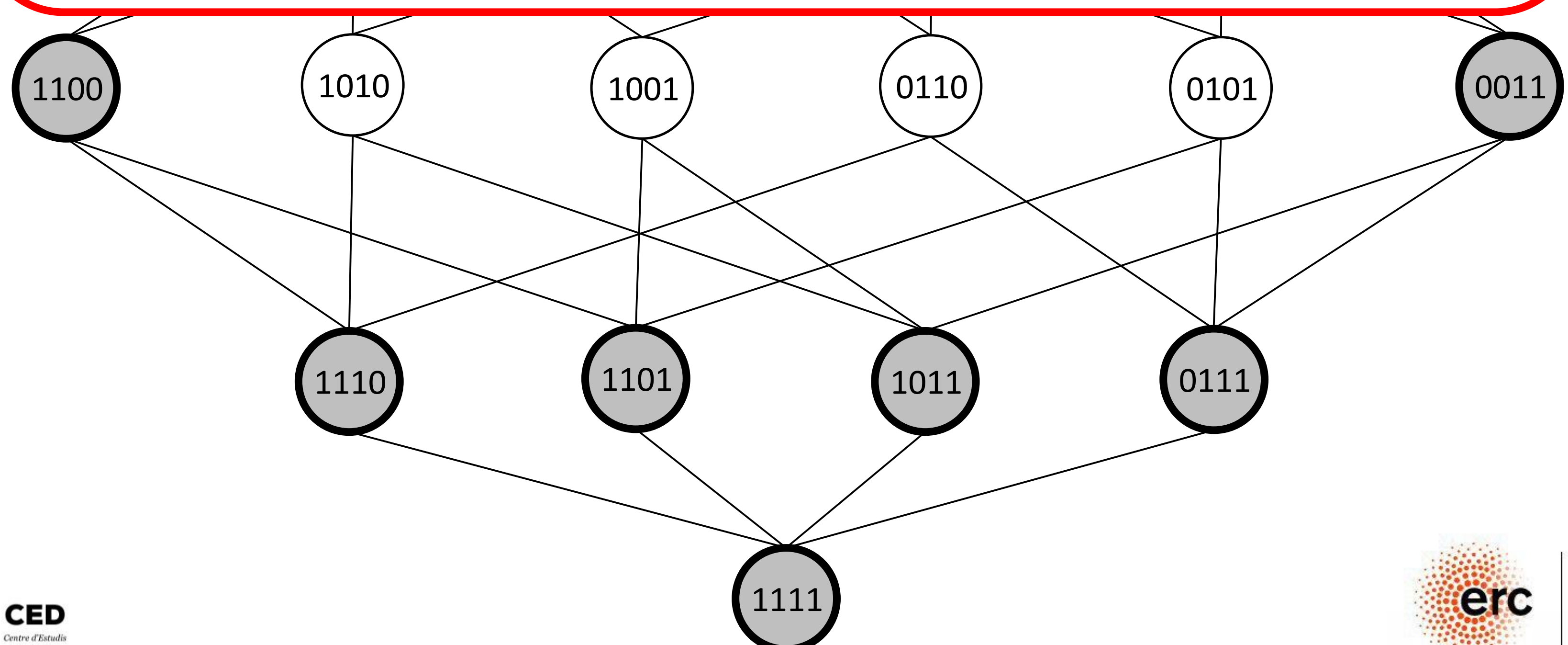
A new 'set of poor profiles' (I)

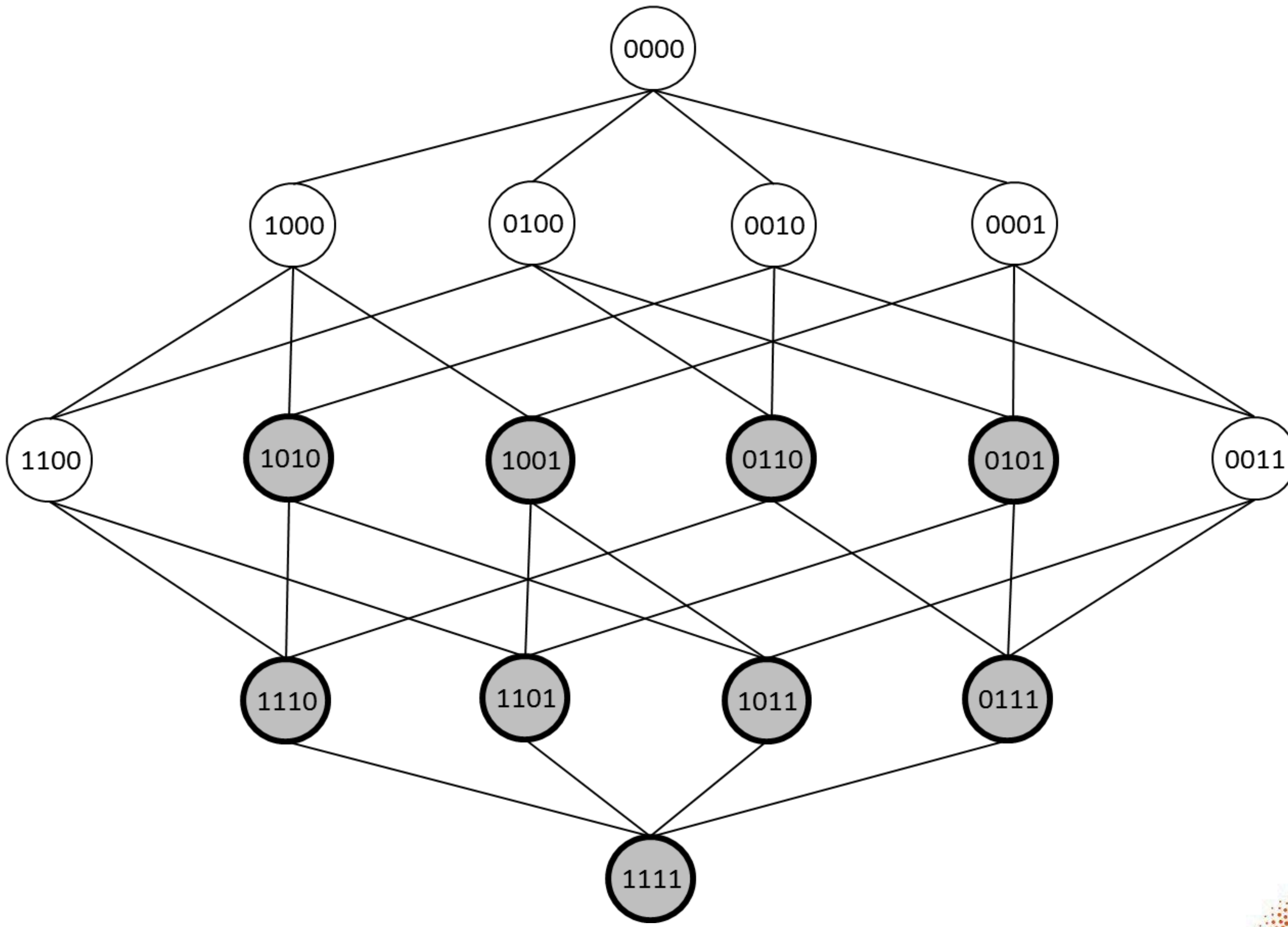


A new 'set of poor profiles' (I)

There exists no weighting scheme (w_1, w_2, w_3, w_4) and no poverty threshold k generating this set of poor profiles via the counting approach



A new ‘set of poor profiles’ (II)



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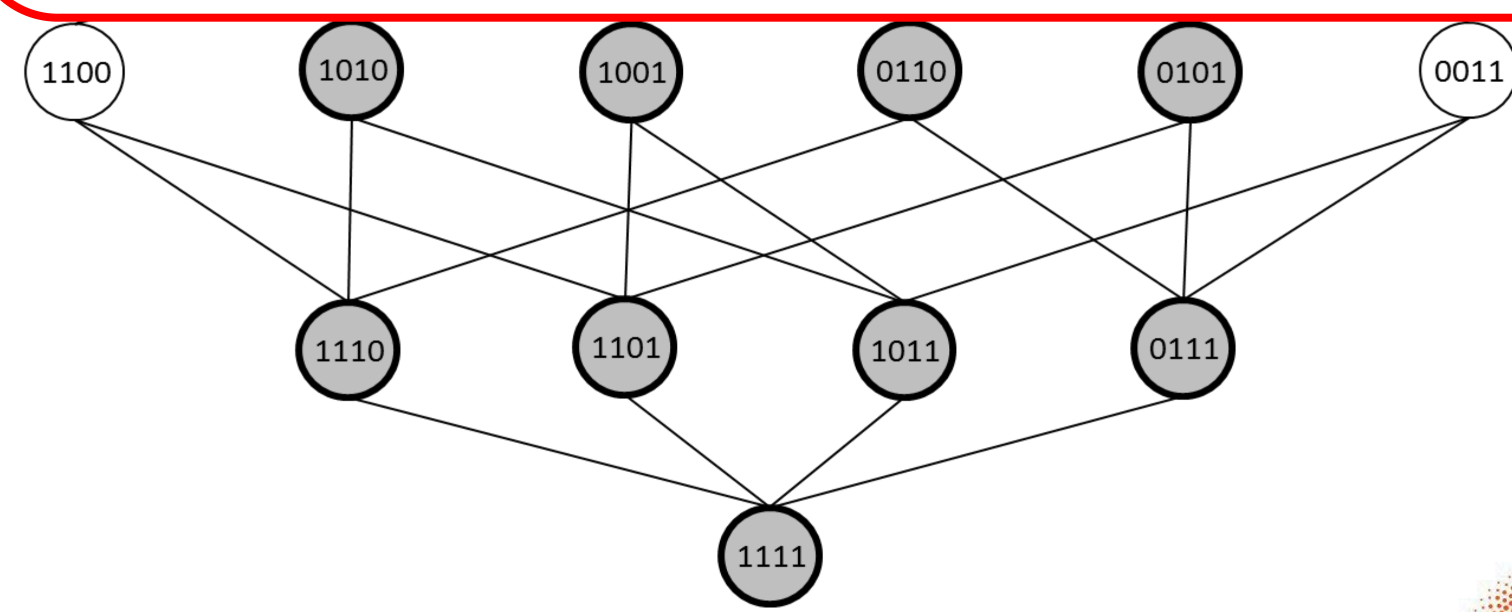

Within & between domain *ρ*-functions

Definition 4: For any natural number $G \leq \lfloor |D| /2 \rfloor$, let $\Pi_{D,G}$ denote the set of partitions of D into G exhaustive and mutually exclusive groups $D_1, \ldots, D_G$ (i.e.: $D_i \cap D_j = \emptyset \forall i \neq j$ and $D = \bigcup_{g=1}^{g=G} D_g$) where each group has at least two members (i.e.: $d_g := |D_g| \geq 2 \forall g$).

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$$\begin{array}{ccc} X^d & \xrightarrow{\rho} & \{0,1\} \\ \downarrow \phi & & \uparrow \rho^b \\ X^{d_1} \times \ldots \times X^{d_G} & \xrightarrow{\rho^w} & X^G \end{array}$$

$$\phi : X^d \rightarrow X^{d_1} \times \ldots \times X^{d_G}$$

$$\rho^w := \left(\rho_1^w, \ldots, \rho_g^w, \ldots, \rho_G^w\right) : X^{d_1} \times \ldots \times X^{d_G} \rightarrow X^G.$$ Within domains

$$\rho^b : X^G \rightarrow \{0,1\}$$ Between domains

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The generalized counting approach

- Specify a poverty threshold within each domain ($m_g \leq d_g$) and a threshold between domains ($M \leq G$). Consider the set of thresholds given by ($m_1, \ldots, m_G; M$).

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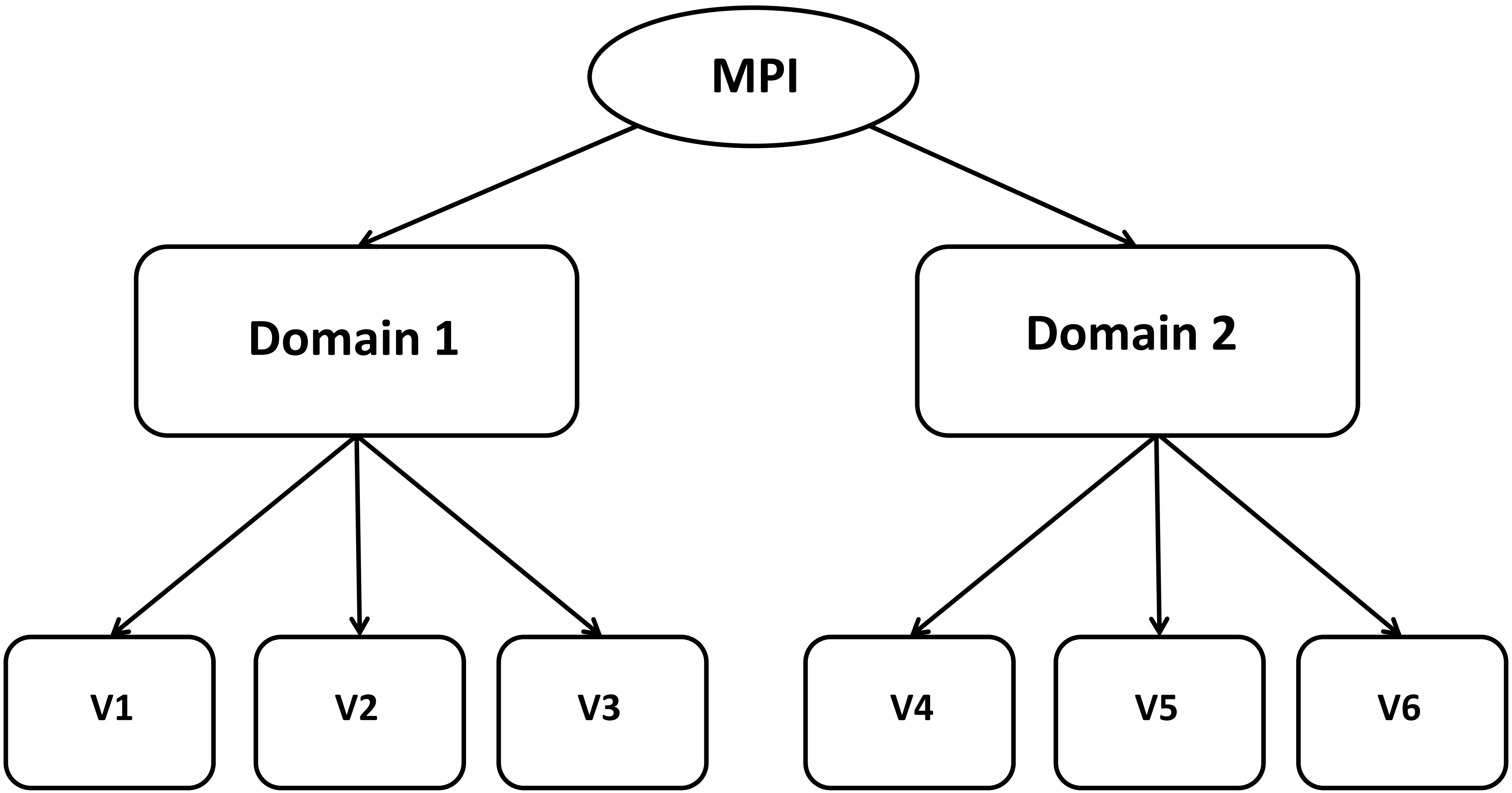
Consider the following sets of conditions

(i) Let $M < G$ and let there be at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ with $m_{g_1} \geq 2$ and $m_{g_2} \geq 2$.

(ii) Let $M = G$ and let there be at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ with $m_{g_1} < d_{g_1}$ and $m_{g_2} < d_{g_2}$.

It turns out that there is no weighting scheme $\mathbf{a}$ and no deprivation score threshold k such that $\rho \in C_d^{\mathbf{a}}$ coincides with the identification functions generated via the generalized counting approach as described in (i) and (ii).

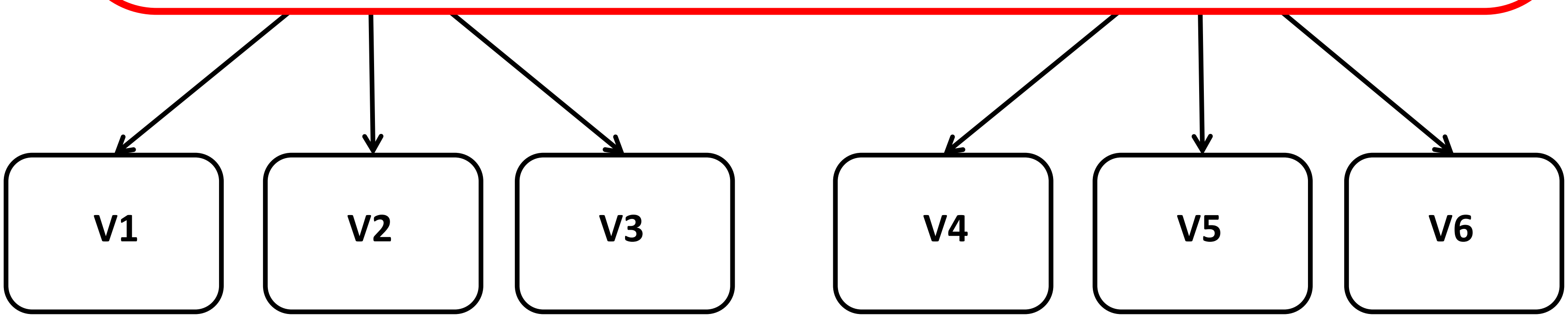
Hierarchically structured indices



Poor identification rule: To be considered as poor, an individual has to be deprived in at least two variables **within** Domain1 **or** Domain2.

Hierarchically structured indices

There exists no weighting scheme (w_1, w_2, w_3, w_4 , w_5 , w_6) and no poverty threshold k generating this set of poor profiles via the counting approach

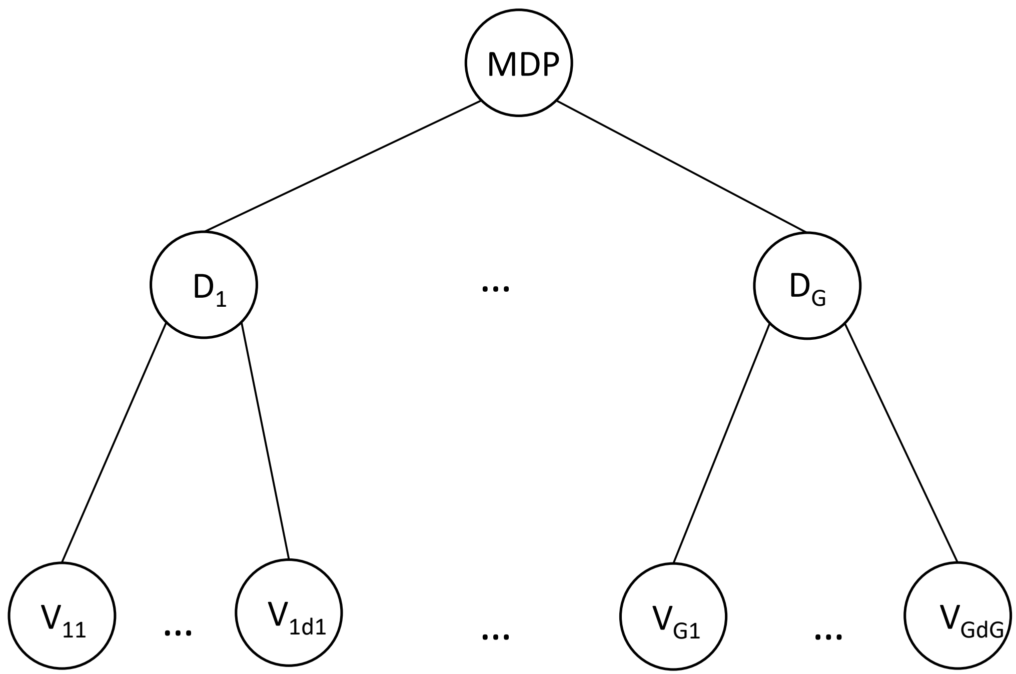


<u>Poor identification rule</u>: To be considered as poor, an individual has to be deprived in at least two variables **within** Domain1 **or** Domain2.

Aggregation in Hierarchically Structured Indices

- Basic indicators structured in domains, sub-domains and so on in a tree-like manner.
  - Example: G domains, with d_g variables in domain 'g'

$\boldsymbol{\gamma}=(\gamma_{11},\ldots,\gamma_{1d1},\gamma_{21},\ldots,\gamma_{2d2},\ldots,\gamma_{G1},\ldots,\gamma_{GdG})$



Aggregation step (1)

- Current approaches: $\mu^{\theta}(\gamma_1,\dots, \gamma_d)$
  - **All pairs of attributes** either complements or substitutes.

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  - **All pairs of attributes** either complements or substitutes.
- New approach: **Domain-first two-stage aggregation**

$$\frac{1}{n} \sum_{i \in Q(P_d)} \Phi\left(\varphi_1(\gamma^c_{i11}, \ldots, \gamma^c_{i1d_1}), \ldots, \varphi_G(\gamma^c_{iG1}, \ldots, \gamma^c_{iGd_G})\right)$$

Allows introducing *within-* **and** *between-domain* elasticities of substitution.

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Aggregation step (2)

$$\Pi_{\boldsymbol{\theta}}(P_d) := \frac{1}{n} \sum_{i \in Q(P_d)} \left(\sum_{g=1}^{g=G} a_g \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{igv}^c)^{\theta_g} \right]^{\theta/\theta_g} \right)^{1/\theta}$$

Proposition 3: Consider the multidimensional poverty measure $\Pi_{\boldsymbol{\theta}}(P_d)$. (i) For any domain D_g ($g \in \{1, \ldots, G\}$), two attributes u, v belonging to that domain (i.e: $u, v \in D_g$) are complements whenever $\theta_g < \min\{1, \theta\}$. On the other hand, the same two attributes are substitutes whenever $\theta_g > \max\{1, \theta\}$. (ii) Assume now the two attributes u, v belong to different domains D_g, D_h ($g, h \in \{1, \ldots, G\}$). Then u, v are complements whenever $\theta < 1$ and substitutes when $\theta > 1$.

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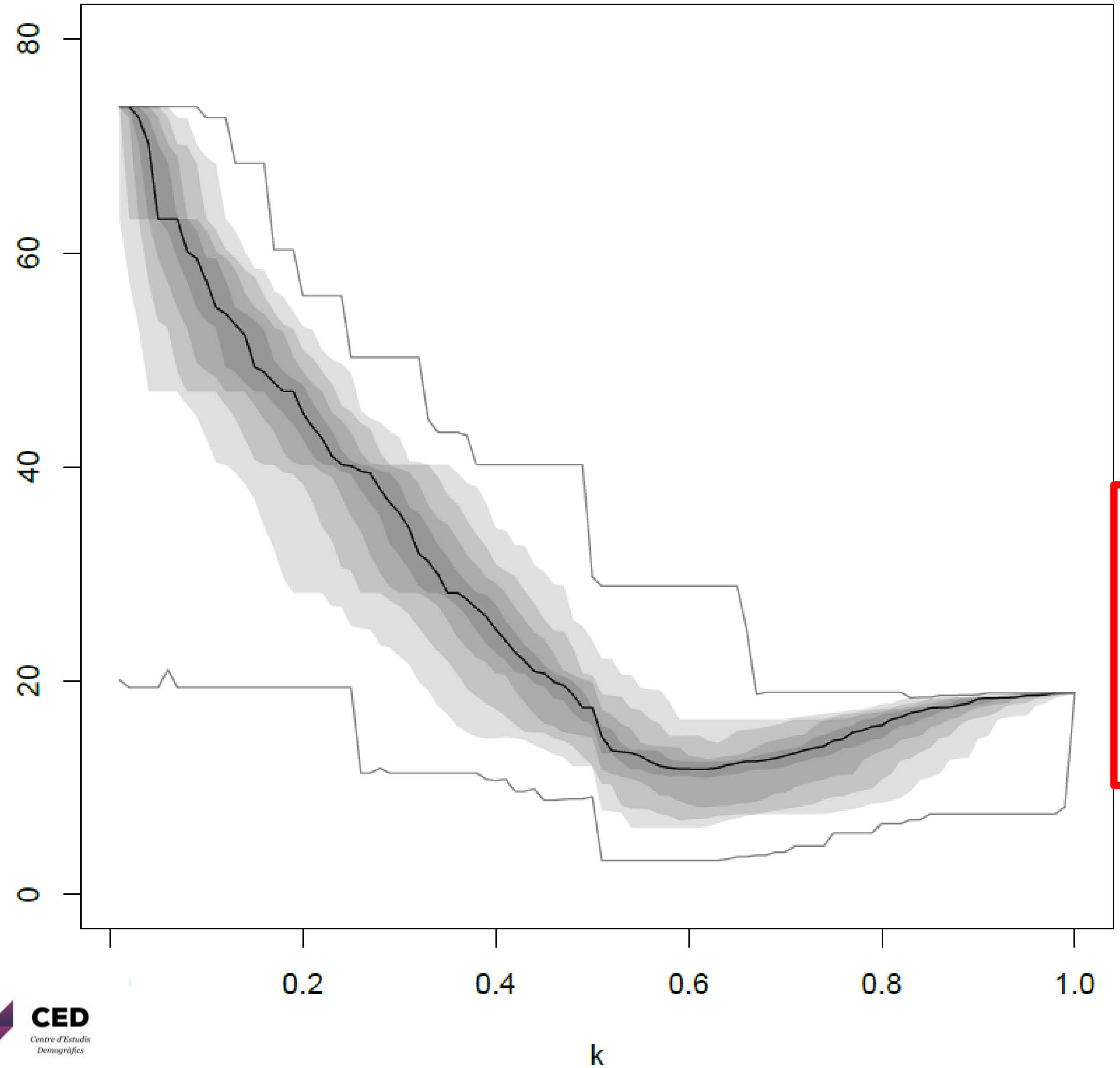
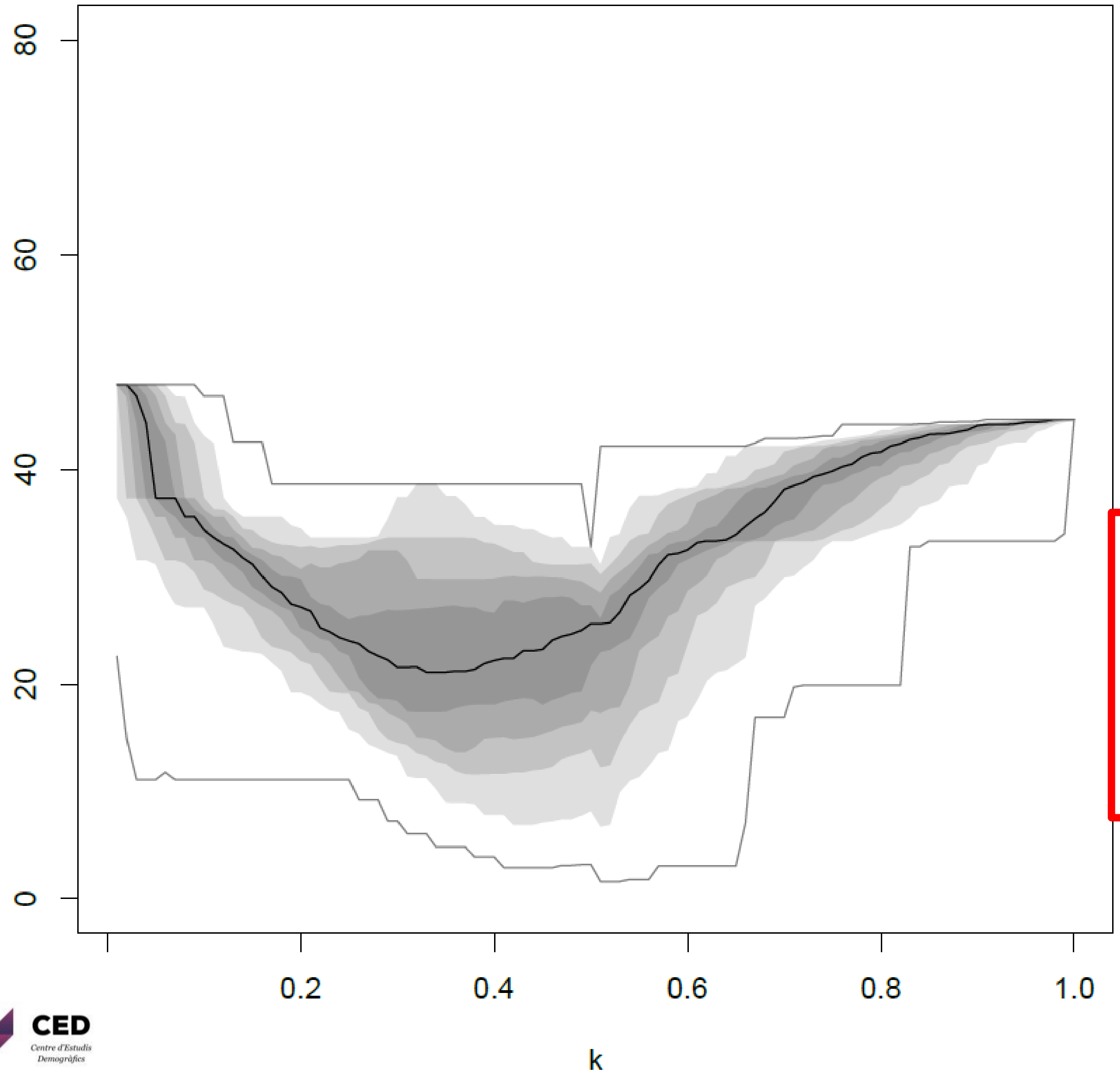
Percentage of misclassified households (1)



Identification function:
Deprived in V1 and V2
or
Deprived in V3 and V4
or
Deprived in V5,V6…,V10

Grand total average:
27% misclassified households

Percentage of misclassified households (2)

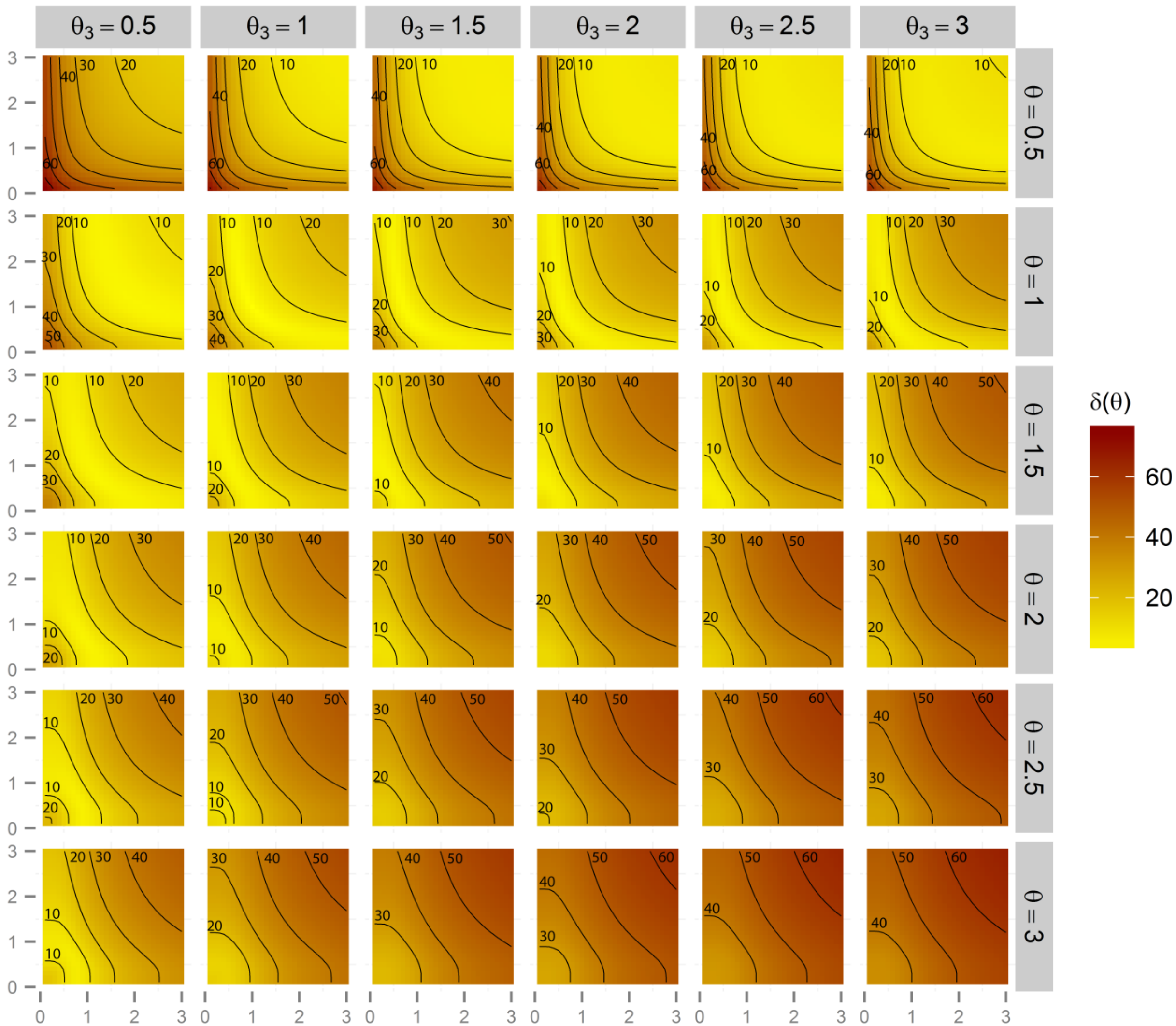


Identification function:
Deprived in V1 and V2
or
Deprived in V3 and V4
or
Deprived in 4 out of the 6 variables: V5,V6…,V10

Grand total average:
32% misclassified households

New aggregation methods

$$\delta(\boldsymbol{\theta}) = \frac{100}{48}\sum_{l=1}^{l=48}\left|1 - \frac{\Pi_{\boldsymbol{\theta}}(P_d)_l}{M_0(P_d)_l}\right|$$



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